termined by measurements of the angle of the oblique shock produced by a small wedge in the plasma stream.

References

[1] Clayden, W. R., "Langmuir Probe Measurements in the R.A.R.D.E. Plasma Jet," *Rarefied Gas Dynamics*, 1st ed., Vol. 2, Academic Press, New York, 1963, Suppl. 2, pp. 435–470.

[2] French, J. B., Sonin, A. A., and deLeeuw, J. H., "The Use of Langmuir Probes in Low Density Plasma Flows," *Rarefied Gas Dynamics*, 1st ed., Vol. 2, Academic Press, New York, 1963, Suppl. 2, pp. 471–494.

[3] Kanal, M., "Theory of Current Collection of Moving Cylindrical Probes," *Journal of Applied Physics*, Vol. 35, No. 6, June 1964, pp. 1697–1703.

[4] Hall, D. F., Kemp, R. F., and Sellen, J. M., Jr., "Plasma-Vehicle Interaction in a Plasma Stream," *AIAA Journal*, Vol. 2, No. 6, June 1964, pp. 1032–1039.

Analysis of Turbulence by Schlieren Photography

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IN this Note we consider the relationship of the statistical parameters that describe the turbulent density variations to the intensity variations on the schlieren photograph of a turbulent flow. We obtain the solution using ray optics. We set the schlieren system with the knife edge oriented parallel to the z axis, and light rays starting in the x direction. The medium through which the rays pass is characterized by an index of refraction, $n = \bar{n} + \mu(x,y,z)$, where μ is a small stochastic quantity describing the variation of the index of refraction from its mean value. Using the equation of the eikonal, the phase variation to the first order of μ is

$$\delta\phi = \int_0^L \mu(x,y,z)dx$$

The integral is to be taken along the straight-line path, $0 \leq x \leq L$, traversed by the rays in the absence of the stochastic variation of the index of refraction. The local deviation of the beam from this straight path (to the first order of μ) is, therefore,

$$\delta\theta = \frac{\delta\phi}{\partial y} = \int_0^L \frac{\partial\mu(x,y,z)}{\partial y}\, dx$$

This expression is the component of angular variation in the y direction (perpendicular to the knife edge). The variation of intensity, $I_1 = \bar{I} - I(y_1,z_1)$, at a point on the image is proportional to $(\delta\theta)$. Thus,[1]

$$I_1 \propto \int_0^L \frac{\partial\mu(x_1,y_1,z_1)}{\partial y_1}\, dx_1$$

The normalized autocorrelation function for the intensities on the photographic plate is now defined as

$$R_{12} = \frac{\langle I_1 I_2\rangle}{\langle I^2\rangle} = \int_0^L \int_0^L \left\langle \frac{\partial\mu(x_1,y_1,z_1)}{\partial y_1}\, \frac{\partial\mu(x_2,y_2,z_2)}{\partial y_2} \right\rangle \times dx_1 dx_2 \Big/ \left\langle \left\{ \int_0^L \frac{\partial\mu(x_1,y_1,z_1)}{\partial y_1}\, dx_1 \right\}^2 \right\rangle$$

Proceeding in a standard way,[2] this result may be written in a more tractable form by introducing the relative coordinates $x = x_1 - x_2$, $y = y_1 - y_2$, $z = z_1 - z_2$ and the definition for the correlation function of the refractive index variations

$$C(x,y,z) = C(x_1 - x_2, y_1 - y_2, z_1 - z_2) = \langle\mu(x_1,y_1,z_1)\mu(x_2,y_2,z_2)\rangle\, \langle\mu^2\rangle$$

where we assume isotropic and homogenous turbulence. Thus,

$$R_{12} = \int_0^L \int_0^L \left[\frac{\partial^2}{\partial y^2} C(x,y,z)\right]_{P_n} dx_1 dx_2 \Big/ \int_0^L \int_0^L \left[\frac{\partial^2}{\partial y^2} C(x,y,z)\right]_{P_n} dx_1\, dx_2$$

where $P_n = (y_1 - y_2)\hat{y} + (z_1 - z_2)\hat{z}$ is the point at which the integrand is to be evaluated. Further simplification can be made by introducing the integral relation

$$\int_0^{L_1} dx_1 \int_0^{L_2} f(x)\, dx_2 = \int_0^{L_1} (L_1 - x)\, f(x)dx + \int_0^{L_2} (L_2 - x)f(x)dx - \int_0^{L_1 - L_2} (L_1 + L_2 - x)f(x)dx$$

If $L_1 = L_2 \gg l$, the scale of turbulence, the upper limit may be replaced with ∞. Thus,

$$\int_0^L \int_0^L f(x)dx_1 dx_2 \doteq 2L \int_0^\infty f(x)dx$$

The final result for the autocorrelation of the intensities is

$$R_{12} = \int_0^\infty \left(\frac{\partial^2}{\partial y_2} C(x,y,z)\right)_{P_n} dx \Big/ \int_0^\infty \left(\frac{\partial^2}{\partial y^2} C(x,y,z)\right)_{P_n = 0} dx$$

In arriving at this final equation, it has been assumed that the path length L traversed by the light rays is large compared to the range of appreciable $C(x,y,z)$.

In order to understand the significance of the result, let the correlation function for the refractive index variations have the Gaussian form

$$C(r) = \exp(-r^2/l^2)$$

Writing the autocorrelation function, it is found that the integrals separate, yielding

$$R_{12} = \frac{\partial^2}{\partial y^2} \exp[-(y^2 + z^2)/l^2]|_{P_n} \Big/ \frac{\partial^2}{\partial y^2} \exp[-(y^2 + z^2)/l^2]|_{P_n = 0}$$
$$= (1 - 2y^2/l^2) \exp[-(y^2 + z^2)/l^2]$$

If positions perpendicular to the knife edge are taken, $z = 0$, $P_n = y$; and the result is

$$R_{12}{}^{\perp} = (1 - 2y^2/l^2) \exp(-y^2/l^2)$$

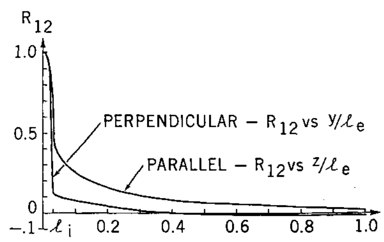

Fig. 1 Autocorrelation for positions parallel and perpendicular to the knife edge.

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Likewise, positions parallel to the knife edge imply that $y = 0$, $P_n = z$; and the result is

$$R_{12}^{\parallel} = \exp(-z^2/l^2)$$

It will be noted that in the perpendicular direction, the autocorrelation for the intensities becomes negative at $y = l/(2)^{1/2}$. This result leads to the expectation that a light area would be followed by a dark area within a distance of the order l. Parallel to the knife edge, the intensities remain positively cross-correlated. This combination of results implies that the photograph would tend to exhibit streaking parallel to the knife edge.

The Gaussian model is a convenient mathematical form. A model which is physically justifiable can be developed in the following way.[3] The index of refraction is a function of temperature; the temperature fluctuations may be regarded as a passive conservative additive in the turbulence. Concentration inhomogeneities appear as a result of the action of the velocity field. Thus turbulent velocity spectra, such as the von Kármán formula, describe the index of refraction in the inertial (outer) range. In the inner range the inhomogeneities are dissipated by molecular diffusion. Thus we consider the following model[3] based upon the von Kármán law:

$$C(r) = \begin{cases} 1 - Ar^2, \; 0 < r < l_i < l_e \\ [2^{2/3}/\Gamma(1/3)](k_e r)^{1/3} K_{1/3}(k_e r), \quad l_i < r < \infty \end{cases}$$

where l_i is the inner scale, and for the present purposes its value is of the order of 1 mm. $l_e = 1/k_e$, the outer scale, is about 3 cm, the size of turbulent wakes for small model experiments. The symbol K represents the modified Bessel function with imaginary arguments.

The constant A is found to be

$$A \doteq 0.952\, l_e^{-2/3} l_i^{-4/3}$$

where we have used the fact that for very small $k_e r$ the second part of the correlation function becomes

$$C(r) \approx 1 - [\Gamma(2/3)/\Gamma(4/3)](k_e r/2)^{2/3}$$

The calculation now involves numerical integration of a rather complicated expression for the second derivative of the correlation of the refractive index variations,

$$\partial^2 C(r)/\partial y^2 = \begin{cases} \{-1.904\, l_e^{-2/3} l_i^{-4/3}\}, \quad 0 \leq r \leq l_i \\ 0.592\, (k_e r)^{1/3} \{ [\frac{1}{3} - 5y^2/9r^2 + \\ \quad \frac{1}{2}(k_e y)^2] K_{1/3}(k_e r)/r^2 + [y^2/3r^2 - 1](k_e/2r) \times \\ \quad [K_{2/3}(k_e r) + K_{4/3}(k_e r)] + (k_e y/2r)^2 [K_{5/3}(k_e r) \\ \quad + K_{7/3}(k_e r)] \}, \quad l_i \leq r < \infty \end{cases}$$

The results of the integration are plotted in Fig. 1 for the case $l_i/l_e = 0.03$. It will be noted that for the von Kármán model the cross correlation in the perpendicular direction again becomes negative, while the parallel correlation remains positive. Thus, the von Kármán model also predicts streaking parallel to the knife edge. In Fig. 2 we plot results for two other cases for different l_i/l_e. Only the perpendicular correlation is shown, since in both cases the parallel correlation remains positive.

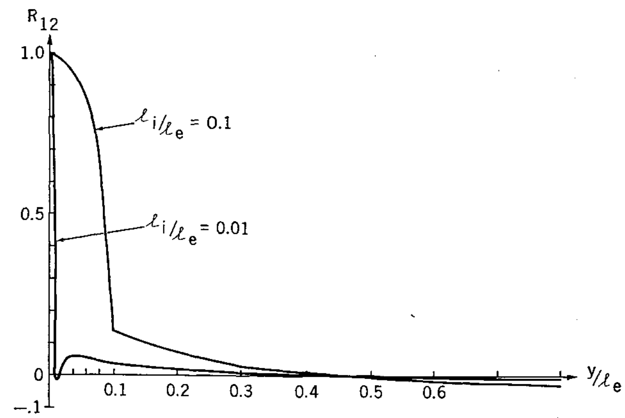

Fig. 2 Autocorrelation for positions perpendicular to the knife edge.

References

[1] Liepmann, H. W. and Roshko, A., *Elements of Gas Dynamics*, Wiley, New York, 1957, pp. 157–160.

[2] Beckmann, P., "Signal Degeneration in Laser Beams Propagated through a Turbulent Atmosphere," *Radio Science Journal of Research*, Vol. 69D, No. 4, April 1965, pp. 629–640.

[3] Tatarski, V. I., *Wave Propagation in a Turbulent Medium*, Pergamon, New York, 1965, Chap. 3.

[4] Gradshteyn, I. S. and Ryshik, I. M., *Table of Integrals, Series and Products*, 4th ed., translated by Scripta Technica Inc., edited by A. Jeffrey, Academic Press, New York, 1965, pp. 1039–1045.

Combustion of an Ablative Fuel in a Constant-Area Channel

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Introduction

At present, a considerable amount of interest has been focused on the feasibility of utilizing ablative materials as fuels for hypersonic airbreathing propulsion systems. The ablative serves both as thermal insulation and as a source of fuel for use in force generation schemes. The primary advantage of this concept is the elimination of problems associated with fuel storage, distribution, and injection. A schematic representation of a solid-fueled ramjet engine is shown in Fig. 1. The solid fuel, which comprises the burner portion of the vehicle, is permitted to ablate, mix with the external air, and burn in the main combustion region. The combustion products can then be expanded to provide thrust.

A recent study[1] has demonstrated the feasibility of burning gaseous ablation products in a high-temperature supersonic airstream. Polymethylmethacrylate (PPM) and nylon models were exposed to high-speed wind-tunnel flows. Resulting combustion of the ablation products was found to be efficient and sustainable, and measurements showed significant combustion-induced pressure rises.

The current interest in the combustible ablator concept has generated a need for additional information regarding the fundamental ablation, mixing, and combustion processes. A program was therefore undertaken to further investigate

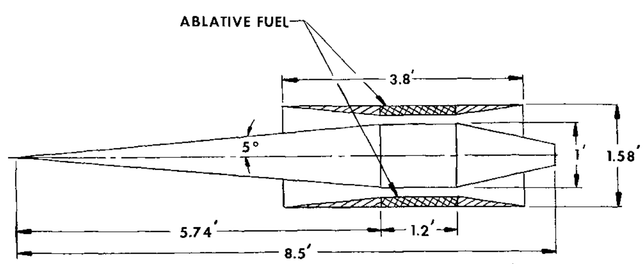

Fig. 1 Schematic of solid-fuel ramjet.

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